UNIVERSITY EXAMINATIONS

EXAMINATION FOR JANUARY/APRIL 2015/2016 FOR BACHELOR OF SCIENCE IN COMPUTER SCIENCE

RCCS 103: CALCULUS (MATHEMATICS).

DATE: 8th/April/2016.   TIME: 2 HOURS

GENERAL INSTRUCTIONS:

Students are NOT permitted to write on the examination paper during reading time.

This is a closed book examination. Text book/Reference books/notes are not permitted.

SPECIAL INSTRUCTIONS:

This examination paper consists Questions in Section A followed by section B.

Answer **Question 1 and any Other Two** questions.

QUESTIONS in ALL Sections should be answered in answer booklet(s).

1. **PLEASE start the answer to EACH question on a NEW PAGE. You will lose 5 MARKS if this is not done.**
2. **Keep your phone(s) switched off at the front of the examination room and NOT on your person.**
3. **Keep ALL bags and caps at the front of the examination room and DO NOT refer to ANY unauthorized material before or during the course of the examination.**
4. **ALWAYS show your working.**
5. **Marks indicated in parenthesis i.e. ( ) will be awarded for clear and logical answers.**
6. **Write your REGISTRATION No. clearly on the answer booklet(s).**
7. **For the Questions, write the number of the question on the answer booklet(s) in the order you answered them.**
8. **Advanced Mathematics log table / Formulas will be provided.**
9. **Calculator will be required.**

SECTION A (COMPULSORY)

QUESTION (1) - (30 MARKS)

a) Define the following terms? **(3Marks)**
   i.   Calculus.
   ii.   Integration.
   iii. Differentiation

b) Evaluate following integrals.

$$\int \frac{x-3}{x^2-x-2} dx \qquad \qquad \textbf{(5Marks)}$$

$$\int_0^\pi \sin(x)\, dx \qquad \qquad \textbf{(3Marks)}$$

c) Compute the limit for the function below. Show the working.

$$\lim_{x \to 3} \frac{x^2-9}{x^2-4x+3} \qquad \qquad \textbf{(5Marks)}$$

d) Given that a line is joined by two points P & Q (given below), **(3 Marks)**
$$P\ (-7,\ 4) \text{ and } Q\ (5,\ -2).$$
Determine the equation of the line perpendicular to line P, Q.

e) Acceleration of an object is expressed using the function below. **(3 Marks)**

$$a = \frac{\Delta v}{t} = 7t^2 + 5t - 3.$$

Find the speed of the object at time, t = 2.3 sec.

f) Distinguish between the definite and indefinite integral **(2 Marks)**

g) Calculate the volume of the solid generated by rotating the curve

$$y = 2x^2 + 3x - 6. \text{ Along the X-axis and bounded by the points}$$

$$(x = 3)\&(x = 5). \qquad \qquad \textbf{(6Marks)}$$

SECTION B (Answer Any Two Questions)

Question (2) - (20Marks)

a) Find the real domain of the following functions. **(4 Marks)**

    i. $f(x) = \dfrac{12x}{x^2 - 36}$

    ii. $f(x) = \dfrac{\sqrt{x+2}}{x-4}$

b) Determine the slope (Gradient) of the following lines. **(4 Marks)**

    i. $y = 10 - 7x.$

    ii. $y = x^2 + 6x - 9.$

c) Integrate the functions below.

(5 Marks)

    i. $\int 3x^2 \sin(x^3 + 1)\, dx$

(4 Marks)

    ii. $\int_0^{2\pi} 2x^3 + 3x - 6\, dx$

d) Determine algebraically, from the first principle, the gradient of the curve below.
$y = 7x^2 + 4x - 3.$ **(3Marks)**

Question (3) - (20Marks)

a) Explain the meaning of following terms. **(5Marks)**

      i.   Derivative.

     ii.   Turning point.

    iii.   Point of inflexion.

    iv.   Maxima.

     v.   Minima.

b) Use logarithmic differentiation to evaluate the derivative of the function below.

(5 Marks)

$$y = \frac{x^2 + 1}{x^3(x-1)^2}$$

c) The following function represents an exponential series.

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} \ldots , \quad -\infty < x < \infty$$

Prove that if $y = e^x$ then $\dfrac{dy}{dx} = e^x$ **(4Marks)**

d) Solve the following integral. Clearly illustrate your solution. **(6 Marks)**

$$\int_3^{10} \sqrt[3]{x-1}\, dx.$$

$$\int sin^2 x\, dx. \quad \text{Given that} \quad sin^2 x = \frac{1}{2}(1 - cos2x)$$

Question (4) - (20Marks)

a) Considering the function below **(8 Marks)**

$$y = \frac{x^3}{2} - \frac{x^2}{4} - 6x + 3$$

Evaluate

   i.   Maximum turning point.
   ii.   Minimum turning point.
   iii.   Inflexion points (if they exist).

b) From the first principle, prove that if, $y = \sin x$ then $\frac{dy}{dx} = \cos x$. **(8 Marks)**

Given that, $\sin \alpha \pm \sin \beta = 2 \sin \left[\frac{(\alpha \pm \beta)}{2}\right] \cos \left[\frac{(\alpha \mp \beta)}{2}\right]$

c) Evaluate the derivatives of the following function.

$$y = 3x^2 \cos 2x.$$ **(4 Marks)**

Question (5) - (20Marks)

a) Draft the graph for the equations/functions below. **(5 Marks)**

     i.   Linear Function $f(x) = x$

     ii.   Parabola $f(x) = x^2$

     iii.   Hyperbola $f(x) = \dfrac{1}{x}$

     iv.   Square Root Function $f(x) = \sqrt{x}$

     v.   Cubic $f(x) = x^3$

b) Solve the following integral. **(4 Marks)**

$$\int_{\pi}^{2\pi} 3x^3 + 5x^2 + 7x + 9 \; dx$$

c) Find the area of the region bounded by the functions below. **(8 Marks)**

$$y = x^2 - 9x + 28$$

$$y = 3x + 14$$

d) Solve the integral function below.

$$\int (5x - 4)^6 \, dx$$ **(3 Marks)**