

# SCHOOL OF EDUCATION

# SEPTEMBER - DECEMBER 2019 END OF SEMESTER EXAMINATION REGULAR PROGRAMME

# EXAMINATION FOR BACHELOR OF EDUCATION (ARTS) REM 414: DIFFERENTIAL CALCULUS

# **INSTRUCTOR: JOHN MBUTHI**

#### DATE: 9<sup>TH</sup> DECEMBER 2019

TIME: 2 HOURS

#### **GENERAL INSTRUCTIONS:**

Students are NOT permitted to write on the examination paper during reading time. This is a closed book examination. Text books/reference books/notes are not permitted.

#### **SPECIAL INSTRUCTIONS:**

- **1.** Write your ADMISSION NUMBER clearly on the cover of the answer booklet(s).
- 2. Answer Question ONE and ANY OTHER TWO questions.
- **3.** Questions in all sections should be answered in answer booklet(s).
- 4. Marks allocated to each question are shown at the end of the question.
- 5. PLEASE start the answer to EACH question on a NEW PAGE.
- 6. Indicate the number of the questions answered on the cover of the answer booklet(s) in the order you answered them.
- 7. Write your answers in paragraph form unless stated otherwise.
- 8. Keep your phone(s) SWITCHED OFF at the front of the examination room.
- 9. Keep ALL bags and caps at the front of the examination room and do not refer to any unauthorized material before or during the course of the examination.
- **10.** You are only allowed to leave the examination room **30** minutes to the end of the Examination.

# QUESTION ONE: COMPULSORY (30 MARKS)

a. Evaluate the following indefinite integrals:

1. 
$$\int 3x(2x^2-5)^2 dx$$
 [5 marks]

2. 
$$\int (x \ln x + \frac{1}{x \ln x}) dx$$
 [5 marks]

3. 
$$\int \frac{\cos^2 x + \sin x}{\cos^2 x} \, dx$$
 [5 marks]

b. Evaluate the following definite integrals:

1. 
$$\int_{ln\frac{1}{2}}^{2} (e^{t} - e^{-t}) dt$$
 [5 marks]

2. 
$$\int_{e}^{e^2} \frac{3}{x \ln x} dx$$
 [5 marks]

3. 
$$\int_0^{\sqrt{\pi}} x \sin\left(x^2 - \frac{\pi}{2}\right) dx$$
 [5 marks]

# **QUESTION TWO**

- a. Find the function f(x) whose tangent has slope  $x^3 \frac{2}{x^2} + 2$  for each value of x and whose graph passes through the point (1, 3). [8 marks]
- **b.** It is projected that t years from now, the population of a certain community in Kenya will be changing at the rate of  $e^{0.02t}$  million per year. If the current population is 50 million, estimate the population of this community after 10 years. [12 marks]

#### **QUESTION THREE**

a. It is estimated that *t* days from now a farmer's crop will be increasing at the rate of  $0.3t^2 + 0.6t + 1$  bushel per day. Determine by how much the value of the crop will increase during the next 5 days if the market price remains fixed at 3 Euros per bushel. ? [12 marks]

# b. Find the area bounded by the curves $y = x^2 - 2x$ and $y = -x^2 + 4$ [8 marks]

#### **QUESTION FOUR**

- a. An object moves along a straight line with acceleration given by  $\frac{dv}{dt} = 1 sin(\pi t)$ . Given that when t = 0, s(t)=v(t)=0, find:
  - i. The velocity function [5 marks]
  - ii. The distance function, of the object. [5 marks]
- b. Find the volume of the solid obtained by revolving the region bounded by  $y = x x^2$  and the x-axis around the *x*-axis. [10 marks]

# **QUESTION FIVE**

- a. Find the arc length of  $f(x) = \frac{1}{3} (x^2 + 2))^{\frac{3}{2}}$  on the interval [0,a] [10 marks]
- b. Compute the area of the surface formed when  $f(x) = y = 2\sqrt{x-1}$  between -1 and 0 is rotated around the x-axis. [10 marks]